Circular TE_{011}-Mode, Trapped-Mode Band-Pass Filters

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Abstract—Band-pass filters are described which utilize a novel type of resonator which has been named a "trapped-mode resonator." This type of resonator uses a structure which is open on its sides so that energy in all but a single desired mode tends to radiate out to energy-absorbing material. However, energy in the desired resonator mode is trapped within the structure and a high-Q resonance occurs, such as is typical of conventional microwave cavity resonators. The use of resonators of this type makes possible the design of band-pass microwave filters which have a pass band similar to that of other multiresonator microwave filters, but without the many unwanted pass bands which are typical of microwave filters. The particular type of filter discussed utilizes the circular TE_{011} mode. Several experimental filter structures of this type were constructed, and the results of laboratory tests are described.

Introduction

Most CAVITY-TYPE band-pass filters have additional pass bands at frequencies that are, roughly, multiples of the primary pass band frequency. In addition, numerous other spurious responses may occur because of resonances involving higher order transverse field variations. Furthermore, in many filter applications, it is desirable to use cylindrical cavities operating in the circular TE_{011} mode, because of the higher Q and power-handling ability this mode affords. Cavities operating in this mode tend to be especially plagued by unwanted resonances. Relatively large, high-Q cavities are essential for narrow-band filters; for this reason, band-pass filters that can eliminate signal components close to the desired fundamental frequency have in the past been unsuitable for suppressing higher order harmonic signals because the large, multimode cavities give rise to many spurious resonances. The novel filter to be discussed avoids this difficulty, so that a single filter might be used to suppress all emissions outside the desired frequency band. Although the principles to be described can also be used with other forms of resonators [1], in this paper the resonators described employ the circular TE_{011} mode which has the advantage of providing a higher unloaded Q than would rectangular TE_{101}-mode resonators.

Operating Principles

Figure 1 shows a possible three-resonator version of the proposed filter connected to circular guides carrying the circular TE_{01} mode. This version of the filter uses resonators composed of "iris plates," with iris slits as shown in Fig. 2(a), alternating with "mode-trap plates" of the form found in Fig. 2(b). The iris plates are spaced $\lambda_{g0}/2$ apart at the band-pass frequency; they form the end walls of each resonator and provide the iris couplings between resonators (see Fig. 1). A mode-trap plate is placed at the center of each resonator, halfway between the iris plates (see Fig. 1), while the outer perimeter of the region between the iris plates and mode-trap plates is filled with lossy material to provide a matched load for any energy that may be radiating between the plates. At the desired resonance, the resonators have a field configuration that is similar to that in a cylindrical resonator operating in the circular (or cylindrical) TE_{011} mode. We have called this type of filter a "trapped-mode" filter because most modes that might occur in such a structure are heavily damped out by the lossy material around the perimeter of the structure, while (for reasons to be explained) a circular TE_{011} mode can be "trapped" in the center portion of the structure so that its energy cannot reach the lossy material. This structure is capable of giving a high-Q, circular TE_{011}-mode resonance in each resonator to yield a good band, but most other mode resonances that could occur in the usual forms of TE_{011}-mode cylindrical resonators are heavily damped or nonexistent.

Note that although this type of filter makes use of lossy load material within its structure, it is not a leaky-wave filter, since leaky-wave filters do not make use of cavity resonances [2]. In contrast, trapped-mode filters use resonances in their pass band and reflect energy at other frequencies. In this filter the lossy material is used to damp out unwanted resonances. At frequencies where unwanted resonances are damped out, the input impedance to the filter will be essentially the reactance of the input coupling iris. Thus, the unwanted signal components are reflected by this filter rather than absorbed. The lossy material may be thought of as causing the coupling into the unwanted modes to be greatly mismatched so that, in fact, very little energy ever does go into the unwanted modes (and into the lossy material). In this respect, the principle is similar to that of the circular TE_{01}-mode waveguide developed by the Bell Laboratories, which uses lossy material interlaced with a metallic helix in the side walls of the guide [3]. (The lossy material in helix guide causes an impedance mismatch for conversion of energy from the TE_{01} mode into unwanted modes and, hence, suppresses the unwanted modes.)

The reason that energy in a circular TE_{011} mode will

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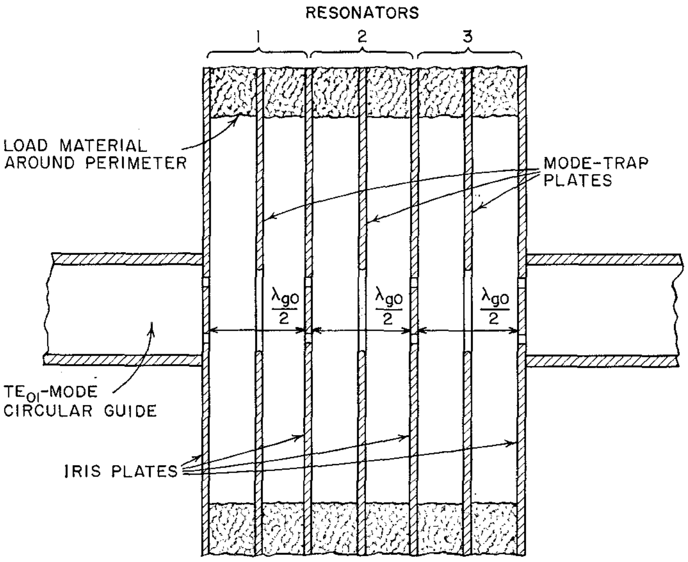


Fig. 1. Three-resonator, trapped-mode band-pass filter connected to circular TE_{01}-mode guides.

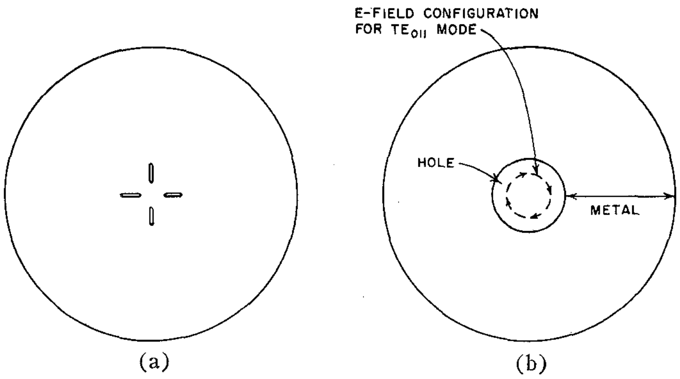


Fig. 2. Iris and mode-trap plates for the filter of Fig. 1. (a) Metal iris plate using slot irises. (b) Typical mode-trap plate.

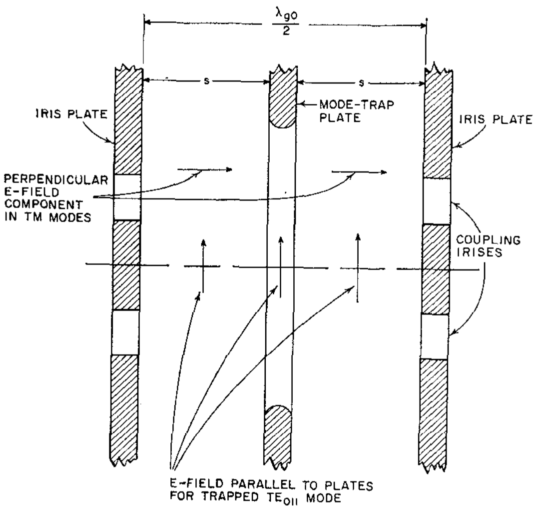


Fig. 3. Enlarged sketch of center portion of a trapped-mode resonator.

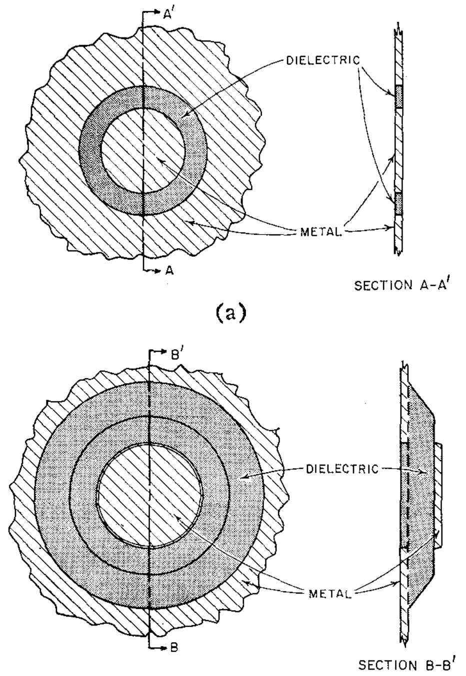


Fig. 4. Two annular iris configurations for possible use in the iris plates of the filter of Fig. 1. (a) Annular iris. (b) Raised annular iris.

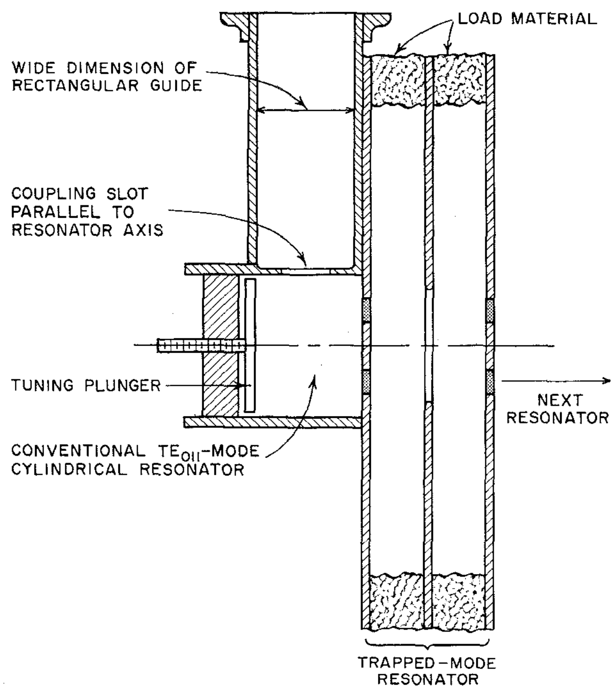


Fig. 5. Use of conventional cylindrical TE_{011}-mode resonator as a transducer for exciting a trapped-mode from rectangular guide.

be trapped in the center portion of the resonators in Fig. 1 can be seen as follows. The E-field for a circular TE_{011} mode has the circular configuration indicated in Fig. 2(b). Thus, as is indicated in Fig. 3, the E-field of the circular TE_{011} mode is always parallel to the metal plates that form the resonator. The metal plates form a radial transmission-line structure, and it is known that a radial transmission-line structure is cut off to radiation having the E-field vector parallel to the plates, unless the plate spacing is a half wavelength or greater. In this case the plate spacings are roughly a quarter wavelength at the pass band frequency, so that the TE_{011}-mode energy cannot escape between the plates.

Although TE_{011}-mode energy is trapped by the radial transmission lines formed by the metal plates, all TM modes present will have a component of E-field that is perpendicular to the plates, as indicated in Fig. 3. The lowest order mode having its E-field perpendicular to the plates in a radial line has no cutoff frequency. Higher order modes having circumferential variations and the E-field perpendicular to the plates do have cutoff frequencies. However, for frequencies such that the resonator could support a TM mode with circumferential variations, the radial line will pass energy. Thus the E-field component perpendicular to the metal plates in a TM resonator mode tends to leak energy from the radial line to the lossy material, and such resonator modes are heavily damped. Of course, other TE resonator modes besides the TE_{011} mode might exist; the radial lines would be cut off to the energy of such modes, up to frequencies where the plates are spaced by a half wavelength or more. However, all TE modes in cylindrical cavities, except for TE_{0mn} modes, require longitudinal wall currents to flow (the TE_{011} mode has only circumferential wall currents). Since no longitudinal currents can flow in the structure in Fig. 1 (i.e., since there are no conductors that permit currents to flow from left to right and back in Fig. 1), it appears that all TE modes except the TE_{011} mode will also tend to be suppressed, at least to some degree. At the second or higher harmonics of the primary pass band frequency, the metal plates are spaced a half wavelength or more apart, so that even the TE_{012} mode is no longer trapped in the structure; and all of the more probable modes should be heavily loaded by the lossy material around the perimeter of the plates. In this manner, strong suppression of practically all (or at least most) of the unwanted resonances that commonly occur in a circular TE_{011}-mode resonator is possible. However, some modes are doubtlessly damped more effectively than others.

Tests, which will be described later in this discussion, indicated that elongated slots such as are shown in the iris plate in Fig. 2(a) tend to generate a small component of electric field in a direction perpendicular to the plane of the metal iris plate. Thus, even if the slots in Fig. 2(a) are driven in a time phase with respect to each other so that the circular TE_{01} mode is generated with maximum efficiency, there is still a small amount of energy present with the electric vector perpendicular to the plane of the iris plates; and this energy will easily radiate between the plates to the lossy load material. This small leakage of energy seriously degraded the unloaded Q of the test resonator (to about 1500) at its primary resonance.

In order to get around the problem previously discussed, the annular iris in Fig. 4(a) was tried. This configuration gave much less leakage, as evidenced by a much higher unloaded Q (around 6400).

The raised annular iris configuration in Fig. 4(b) is also of interest. It has many of the same properties of the planar annular iris, but having the metal center disk raised from the plane of the iris plate may have some advantage in that it may help to guide the energy in unwanted modes out to the lossy load material at the perimeter of the resonator. The sizable disk of dielectric material shown was found to be desirable from the standpoint of increasing the coupling of the iris without having to raise the center metal disk an unreasonable amount. The dielectric material should also help to guide energy at high frequencies to the load material at the perimeter of the resonator. At the fundamental resonance, the dielectric material is in a region of relatively low electric field so that the dielectric losses should not be large. This is especially true if materials such as Tellite or Rexolene P are used, which have dissipation factors of less than 0.0001.

The trapped-mode resonators under discussion are most easily driven by cylindrical terminating guides carrying the circular TE_{01} mode. However, these resonators can also be made use of when the driving system uses rectangular guide. Figure 5 shows what is probably the easiest method for obtaining from a rectangular guide system the circular TE_{01}-mode driving signal that is required in order to excite properly a trapped-mode resonator at its fundamental resonance. The circuit shown uses a conventional cylindrical TE_{011}-mode resonator [4] as a transducer for obtaining the desired circular TE_{01}-mode exciting fields. The TE_{011}-mode resonance in the cylindrical resonator is excited by a longitudinal slot in the end wall of the rectangular input guide. Using this approach, a multiresonator filter might use a conventional TE_{011}-mode cylindrical resonator at each end and trapped-mode resonators in the interior of the filter. The cylindrical and the trapped-mode resonators would then all contribute to the sharpness of the cutoff adjacent to the pass band, while the trapped-mode resonators would help to insure a broad stop band that is free of unwanted pass bands.

In Fig. 5 a tuning plunger is used that does not make contact with the side walls of the cavity. This is feasible, of course, since the wall currents in TE_{011}-mode resonators are all circumferential in direction. In addition, many spurious modes in this type of cavity can be reduced in strength by adding some lossy material behind the tuning plunger [4].

Determination of the Resonant Frequency

The resonant frequency of a cylindrical resonator operating in the TE_{011} mode is given by the formula [4]

$$f = \frac{v}{D}\sqrt{1.49 + \frac{1}{4}\left(\frac{D}{L}\right)^2} \qquad (1)$$

where v is the velocity of propagation in the medium within the resonator, L is the length of the resonator, and D is its diameter. In the case of the trapped-mode configuration in Fig. 3, the metal side walls of a conventional TE_{011}-mode cylindrical resonator are replaced by what amounts to two radial lines below cutoff. The electric field vector of the TE_{011} mode is parallel to the plane of the metal plates, and we have a situation similar to a certain configuration that is treated in [5]. There the equivalent circuit for a wave impinging on an infinite array of plates is given for the case where the E-field is parallel to the plates and the plates are so closely spaced that all propagation with the E field parallel to the plates is cut off in the region of the plates. Marcuvitz [5] shows that the equivalent circuit for this situation is simply a transmission line with a short circuit located at some distance back from the leading edge of the plates. A similar situation must hold for the circuit in Fig. 3 when it is resonating in the TE_{011} mode, since this mode has its E-field parallel to the plates. The radiation of the TE_{011} mode is blocked at the edge of the hole in the mode-trap plate because, due to the close spacing of the plates, the propagation between the mode-trap and iris plates is cut off. This cutoff effect at the edge of the hole in the mode-trap plate has an influence on the fields in the interior of the resonator which is much the same as if the mode-trap plate were removed and instead a cylindrical metal wall enclosed the fields. The diameter of this equivalent cylindrical metal wall is, of course, somewhat larger than the diameter of the hole in the mode-trap plate.

By use of the data in [5], it is possible to estimate how far out from the edge of the hole in the mode-trap plate the equivalent cylindrical metal wall would be. This then gives an equivalent diameter D for the trapped-mode resonator. The length of the resonator L is measured from the inner face of one iris plate to the other iris plate. Then using (1), an approximate resonant frequency for the resonator can be computed. As the experimental data about to be discussed will show, this approximate method for computing the resonant frequency of mode-trap resonators gives surprisingly good results.

Experimental Results and Further Insights

Single-Resonator Tests

Various tests were made on single, trapped-mode resonator configurations [6], where the resonator was excited by a conventional form of rectangular-waveguide-to-circular-TE_{011}-mode waveguide transducer. One interesting result of these tests was the good agreement between the computed resonant frequencies obtained as described above and the measured resonant frequencies of the trapped-mode resonators. Some of these results are tabulated in Table I. Considering the approximations involved, the agreement is found to be quite good. The last case listed in the table is a case where, instead of using only one mode-trap plate, two mode-trap plates were used between the iris plates. This arrangement was found to give poor rejection of higher order resonances and was found to be undesirable except possibly for cases where the desired primary resonance is to have a higher order field variation along the longitudinal axis of the resonator.

TABLE I

Calculated and Measured Fundamental Resonant Frequencies for a Trial Trapped-Mode Resonator

Calculated f_0 (Gc)	Measured f_0 (Gc)	Plate Spacing (See Fig. 3)
8.52	8.631	0.544
8.81	8.817	0.512
9.00*	9.087*	0.256*

* In this case two mode-trap plates were used instead of one. This gave a stack of four plates: an iris plate, two mode-trap plates, followed by another iris plate, all spaced 0.256 inch apart.

One single resonator was tested using iris plates with four slots as shown in Fig. 2(a). The unloaded Q of this resonator was found to be disappointingly low (about $Q_u = 1500$). Tests on this resonator made with a small probe showed that the E-field between the metal plates had an appreciable component which was vertical to the metal plates. Since a pure TE_{011} mode would have all of the E-field oriented parallel to the metal plates, and since any E-field component which was perpendicular to the plates would tend to propagate between the plates to the load material around the perimeter of the resonator, this component of E-field vertical to the metal plates could very easily account for the low unloaded Q observed for this test resonator. Further probing of the electric field showed that there was an eight-lobe radiation pattern observable as the probe was moved around the perimeter of the circular plates. The symmetries of this eight-lobe pattern lined up with the symmetries of the four-slot iris configuration and indicated that the iris slots themselves were launching radiation having an E-field component perpendicular to the metal plates. For this reason it was decided that the type of iris plate shown in Fig. 2(a) which uses radial slots is undesirable.

In order to avoid the disadvantages of the radial-slot type of coupling, it was decided to try an annular-slot coupling arrangement such as shown in Fig. 4(a). This type of iris has perfect circular symmetry; and if a perfectly circular TE_{01} mode impinges upon this iris, there is no reason why anything but a perfect TE_{01} mode should be radiated from this iris. Tests on a single trapped-mode resonator using this type of iris gave considerably improved results. The unloaded Q went up from 1500 to 6400. Tests were made on this resonator from 7.4 to 22 Gc, and most spurious responses appeared to be attenuated by 25 dB or more. However, since the rectangular and circular waveguide transducer used with this resonator was of relatively narrow bandwidth [7], it was very difficult to say how much of the transmission characteristic was due to the resonator and how much was due to the transducers at the ends of the resonator.

Tests were also made on a modified version of this single resonator, which used a raised annular iris as shown in Fig. 4(b). The idea of the raised iris was that it would tend to channel energy at high frequencies towards the load material around the perimeter of the trapped-mode resonator. Thus it was anticipated that the use of this type of iris would give higher attenuation of any possible spurious responses at high frequencies. Tests made on this resonator configuration tended to confirm this idea. Using an experimental setup which had a sensitivity of about 30 dB, no spurious responses were observed above 14.4 Gc when tests were made from 7.4 to 22 Gc.

Three-Resonator-Filter Tests

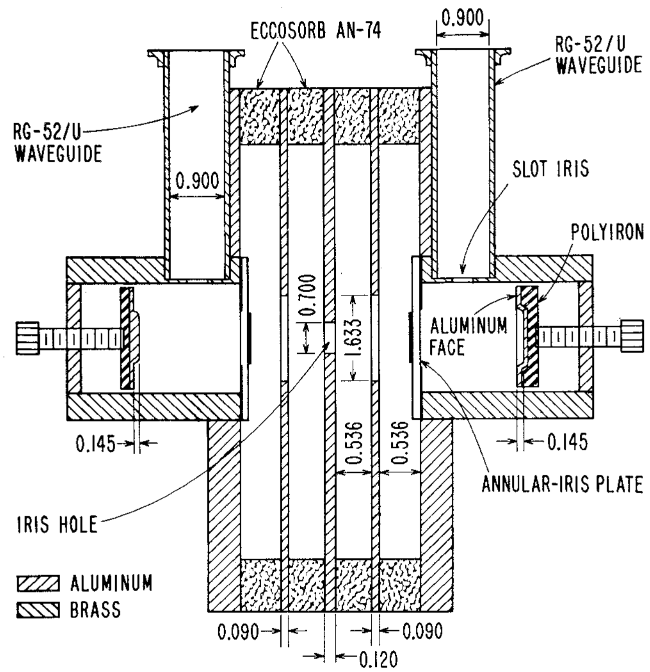
Next a three-resonator filter of the general form in Fig. 5 was constructed. This filter used a conventional circular TE_{011}-mode resonator at each end of the structure and a single trapped-mode resonator in between. Thus, the conventional resonators at the ends of the filter served as rectangular-waveguide-to-circular-waveguide transducers as well as resonators for the filter. The end resonators would, of course, be subject to all of the numerous spurious resonances that are typical of TE_{011}-mode resonators. (However, the resonators did use tuning plungers with a gap around their perimeter along with Polyiron backing, which helps to reduce some of the spurious responses [4].) The trapped-mode resonator in the center served as an added resonator for the pass band of the filter while also providing a means for eliminating, or at least attenuating, spurious responses.

Tests on this three-resonator filter were reasonably encouraging. Nearly all of the spurious responses observed had a minimum attenuation in the 25 to 30 dB range. However, there tended to be two rather pronounced spurious responses, one at 7.84 Gc, and the other at 9.91 Gc (while the pass band was at 9.22 Gc). Some study of this matter indicated that the response at 7.84 Gc was due to a TE_{211}-mode resonance in the end cavities; and this theory was further substantiated by the fact that if the input and output resonators were so oriented that the axes of the input and output rectangular waveguides made an angle of 45 degrees with each other, the spurious response at 7.84 Gc was eliminated. Study of the field pattern of a circular TE_{211} mode will show that if an iris slot excites the TE_{211} mode in one resonator, the output iris slot will not couple to the fields if the output iris slot is moved around the circumference of the resonator by an angle of 45 degrees from that of the first resonator. Similarly, tests suggested that the fairly large response at 9.91 Gc (which had a minimum attenuation of about 13 dB) was due to a TE_{311}-mode resonance in the end cavities. This idea was substantiated by the fact that if the end cavities were rotated about their longitudinal axes so that the input and output rectangular waveguides made an angle of 90 degrees with respect to each other, this response at 9.970 Gc was no longer observed. Study of the field configuration of the TE_{311}-mode resonance will show that if the slot iris in one resonator excites a TE_{311}-mode resonance and this resonance excites a TE_{311}-mode resonance in the cylindrical resonator at the other end of the filter, the output iris slot will not couple to the fields in the output resonator if the iris position on the cavity wall is 90 degrees with respect to the position of the iris slot on the input resonator. Therefore, no spurious transmission will be observed.

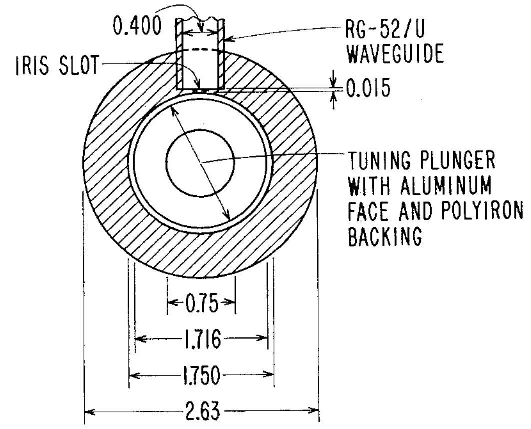
The results of these various tests on the three-resonator filter of the general form in Fig. 5 indicated that the spurious responses present were largely due to resonances in the cylindrical end cavities of the structure. When these resonances occurred, the trapped-mode resonator was probably not resonant, and it provided considerable attenuation of the unwanted spurious response. The power that did get through appeared to result from coupling directly from the annular iris on the cylindrical cavity on the left directly to the annular iris for the cylindrical cavity on the right. One way of reducing this coupling would be to add an additional trapped-mode resonator so that these two annular irises would be further separated and more greatly isolated. For this reason it was decided to add one more resonator to the structure and to make it into a filter with two conventional circular TE_{011}-mode resonators and two TE_{011}-mode, trapped-mode resonators. (As the results described later will show, another effective way of solving this problem would have been to use cylindrical end cavities having somewhat different proportions so that when their desired TE_{011}-mode resonances coincide, their numerous other unwanted resonances would fall at somewhat different frequencies. Then when one resonator is exhibiting a resonance at an unwanted mode, the other resonator will not be resonant, and the trapped-mode resonator in between will provide resistive padding between the two reactive resonators. In this way high rejection can be obtained.)

Four-Resonator-Filter Tests

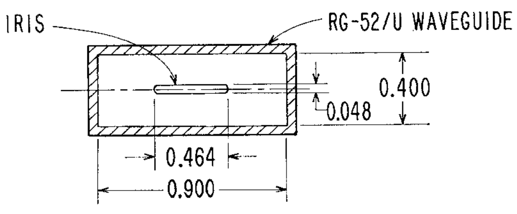
Figure 6 shows the approximate form of the trial four-resonator trapped-mode filter, and Fig. 7 illustrates the filter. Note that in this case two trapped-mode resonators are used instead of one. The coupling between the two trapped-mode resonators is accomplished in this case by a simple round hole, rather than by use of an annular iris. This was done because it was felt that the use of a round hole between the two trapped-mode



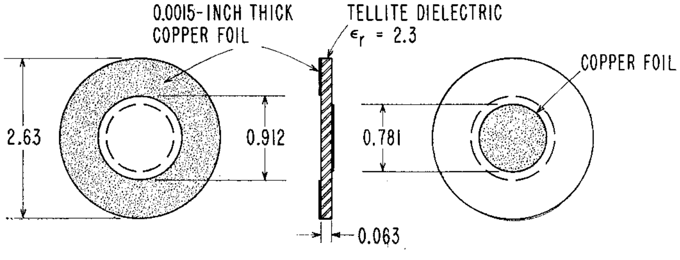
(a) Trial, four-resonator circular TE_{01}-mode filter using two trapped-mode resonators and two conventional resonators. The pass-band frequency is approximately 9.2 Gc.



(b) End cavity cross section.



(c) Slot iris detail.



(d) Annular-iris plate.

Fig. 6.

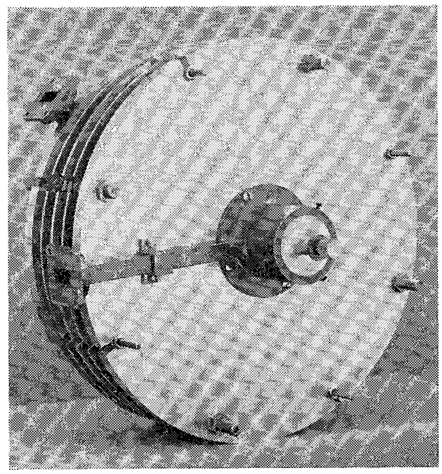


Fig. 7. Trial filter using two conventional and two trapped-mode circular TE_{011}-mode resonators. The plates for the trapped-mode resonators were oversized to insure plenty of space around the perimeter for load material and for probing of the fields.

resonators would provide greater isolation between the annular irises than would an additional annular iris in the center of the filter. At high microwave frequencies, the upper stop band attenuation is sometimes degraded by a "beam" effect; undesired energy couples directly from the input port to the output port of a multiresonator filter. The combination of the small round iris with the two annular irises (Fig. 6) tends to block the direct transmission of energy, creating a broad upper frequency stop band. In addition, the use of a simple round hole made the structure easier to fabricate. In the drawing it will be noted that the tuning plungers in the cylindrical end cavities are somewhat different for one cavity than for the other. In the first model this was not the case and the tuning plungers had identical flat faces. The reason for the change to tuning plungers whose faces were not flat and which were different for the two end cavities will be discussed later.

Preliminary tests were made on various parts of the filter structure in order to predetermine the proper size for the various iris couplings. It was desired that the filter response correspond approximately to that obtainable from an equal-element low-pass prototype filter [8], [9]. By laboratory techniques, the external Q of the end resonators and the coupling coefficients between resonators were measured [10] and adjusted to achieve approximately the desired values. The external Q of one of the end resonators was determined by detuning all of the other resonators in the filter by detuning the plunger on the other end cavity, and by inserting metallic objects into the two trapped-mode resonators. Then the external Q of the remaining end cavity was determined by standard techniques using a waveguide slotted line [10].

Before the coupling coefficients between resonators could be checked, it was necessary to tune up three of the resonators of the filter. This was accomplished by using a sweeping signal generator, a reflectometer at the input to the filter, and an oscilloscope to display the reflected-power-vs.-frequency characteristic. The general principle used is as follows. Using the tuning plunger on the left, the end cavity was tuned to the desired frequency while all of the other resonators of the filter were detuned. Observing the display on the oscilloscope reveals a single dip in the reflected power-vs.-frequency characteristic. Next, the second resonator was tuned into synchronism with the first resonator by adjusting the spacing between the iris plates of the second resonator, making use of the adjustable bolts at the perimeter of the resonator. It can be shown that in this situation the second resonator is properly tuned when the reflected energy-vs.-frequency characteristic on the oscilloscope displays a *symmetrical* double-humped response, with the center line of the response pattern located at the same center line as the center line of the response for the first resonator by itself. (A synchronously tuned band-pass filter will have a symmetrical pass band response when properly tuned. Of course, due to the variation of the coupling coefficients with frequency, this response will be slightly skewed, but the symmetry which results from proper tuning is usually easy to discern.) Finally, to tune the third resonator of the structure, the metallic detuning material is removed from the third resonator and the spacing between the iris plates of the third resonator was adjusted until the reflected power-vs.-frequency pattern on the oscilloscope showed a symmetrical pattern having three dips, where the center dip was located at the same point where the single dip for the first resonator alone had appeared. In this way, the first three resonators of the filter were synchronously tuned, while the fourth resonator was left detuned.[1]

In order to measure the coupling coefficient between the first and second resonators, a piece of metal was inserted into the third resonator in order to detune it. Next a small coupling probe was inserted far enough into the second resonator so that light coupling was obtained to the field in the resonator at resonance. Then the transmission characteristic from the input waveguide to the coupling probe in the second resonator was noted, and two, separate, strong peaks of transmission were noted, as expected. The frequency spacing Δf_m between these two peaks of transmission was measured, and the fractional bandwidth

$$w_m = \frac{\Delta f_m}{f_0} \tag{2}$$

was computed, where f_0 is the center frequency halfway between the two peaks of transmission. By adaptation of a formula in [10], the coupling coefficient between the resonators was computed using

$$k_{12} = \sqrt{w_m^2 + \frac{1}{4}\left(\frac{1}{Q_{e1}} + \frac{1}{Q_{e2}} + \frac{1}{Q_{u1}} + \frac{1}{Q_{u2}}\right)^2}. \tag{3}$$

[1] This tuning procedure is closely related to the tuning procedure described by Dishal [12].

In this equation, Q_{e1} is the external Q of the first resonator, while Q_{e2} is the external Q of the second resonator (due to loading by the coupling probe). Similarly, Q_{u1} is the unloaded Q of the first resonator, and Q_{u2} is the unloaded Q of the second resonator. Values of Q_{e1} and Q_{u1} were obtained by measurement [10], and values for Q_{e2} and Q_{u2} were estimated. Since the term involving Q's in (3) is relatively small, the estimated values for Q_{e2} and Q_{u2} need not be very accurate.

Measuring the coupling coefficient between Resonators 2 and 3 poses somewhat of a problem inasmuch as means must be provided for exciting circular TE_{011}-mode resonances in the resonators. Reasonably good results were obtained by first having Resonators 1, 2, and 3 synchronously tuned, and then detuning Resonator 1 until it was so far off resonance that it had little effect on the shape of the response in the vicinity of the frequency of synchronous tuning f_0 but yet would still launch energy in a reasonably pure TE_{01} mode. Thus, with Resonator 1 detuned so as to give very loose coupling at the input of the filter and with the coupling probe moved to Resonator 3 to give loose coupling to Resonator 3, transmission measurements were again made and the frequency spacing Δf_m between the two peaks of transmission observed was measured. With the use of this technique, the coupling coefficient was computed by use of (3) with the subscripts 1 and 2 changed to 2 and 3, respectively.

After some adjustment of iris sizes, the external Q_e for the end cavities for this filter was approximately 1145. For a band-pass filter designed from an equal-element low-pass prototype [8], [9], the coupling coefficients between the resonators should be $k_{12} = k_{23} = k_{34} = 8.72 \times 10^{-4}$, to go with $Q_e = 1145$ [10]. The values for the coupling coefficients obtained from the measured data were $k_{12} = 9.64 \times 10^{-4}$, $k_{23} = 8.57 \times 10^{-4}$, and by symmetry k_{34} should equal k_{12}. These values were considered to be close enough to the desired values. In computing the coupling coefficients from the measured data, the unloaded Q's were assumed to be approximately 7500, and the loading due to the sampling probe was assumed to be negligible (which implies that the external Q of the resonator with the sampling probe in it, is infinite).

The entire filter was tuned using a sweeping signal generator and an oscilloscope, in a manner as described previously; and then transmission tests were made on the overall structure. The results are shown in Table II. It should be noted that these results are for an earlier version of the structure in Fig. 6. This version used identical flat faces on the tuning plungers in the end cavities instead of the tuning-plunger faces shown in the figure. By very careful adjustment of the filter, the insertion loss in the pass band was as low as 1.8 dB; but for the tuning adjustments for which the data in the table were taken, the minimum insertion loss was 2 dB. For the case of 1.8 dB insertion loss, by working backward from the measured response, the estimated average unloaded Q for the resonators was approximately 8200. As is seen from the table, the strongest spurious response observed outside of the pass band had a minimum attenuation of 28 dB. This performance seemed reasonably good, but efforts were continued to see if even higher rejection of spurious responses could be obtained.

TABLE II

MEASURED PERFORMANCE FOR FILTER IN FIG. 6 WHEN USING IDENTICAL, FLAT, TUNING PLUNGERS IN THE END CAVITIES*

Frequency (Gc)	Minimum Attenuation (dB)	Notes
9.26	2.0 (As low as 1.8 dB with precise-tuning.)	3-dB bandwidth ≈ 13.5 Mc 20-dB bandwidth ≈ 19.9 Mc 30-dB bandwidth ≈ 25.4 Mc
13.51	35.2	Spurious response
17.00	45.1	Spurious response
17.60	37.5	Spurious response
18.67	28.0	Spurious response
19.78	39.5	Spurious response

* Measurements were made from 7.4 to 22 Gc, and *the spurious responses listed are those noted which had a minimum attenuation of approximately 45 dB or less.* The attenuation was extremely high at frequencies between the various responses. The input and output guides were oriented at 45° with respect to each other.

It was believed that the spurious responses that were detected were due to resonances in the end cavities alone, and that these resonances were being heavily attenuated by the trapped-mode resonators. It appeared likely that if the two sets of unwanted resonances in the two end cavities could be made to occur at different frequencies, the attenuation of the unwanted spurious responses would become extremely high. It should be noted that for an ordinary, multicavity filter, designing the various resonators so that their higher resonances occur at different frequencies is usually not very effective. This is because conventional multicavity filters are purely reactive structures, and if any one of the resonators becomes resonant, it tends to drag the entire structure into resonance and a strong spurious response will result. The filter in Fig. 6(a) differs from a conventional multicavity filter inasmuch as at frequencies off the pass band frequency, the trapped-mode resonators tend to provide resistive padding between the conventional cavity resonators at the ends of the filter. Because of this resistive padding effect, a resonance in one resonator cannot force the entire structure into resonance. Thus if the unwanted resonances in the end cavities are at somewhat different frequencies, very high overall attenuation should result.

In order to make the unwanted resonances of the end cavities occur at somewhat different frequencies, it was decided to make the resonators somewhat dissimilar. The easiest way to do this appeared to be to alter the shape of the faces of the tuning plungers in the end resonators as shown in Fig. 6(a). The tuning plunger on the left has a cylindrical disk protrusion on its face, while the tuning plunger on the right has a cylindrical recess cut in its face. Table III shows the measured performance of this filter with these modified tuning plungers. Efforts were made to increase the sensitivity of the test system so that spurious responses having minimum attenuation of roughly 60 dB or less could be detected. As will be seen from the results in Table III, the minimum attenuation of all spurious responses detected was 44 dB or more, and most responses were attenuated by 55 dB or more. Note that the input and output waveguides were oriented at 45 degrees (see Fig. 7), which would tend to discriminate against any transmission due to the TE_{211} mode. Whether this was necessary or not in this case was not investigated. It is very probable that with dissimilar resonators at the ends of the filter, such measures will not be necessary. All in all, the attenuation of unwanted resonances in this filter is considerably higher than that achieved by the previous techniques for suppression of unwanted resonances in filters using the circular TE_{011} mode. (A carefully designed filter using the various previous techniques is the filter of E. M. T. Jones described in [13].)

TABLE III

MEASURED PERFORMANCE FOR THE FILTER IN FIG. 6 WHEN USING DISTORTED TUNING PLUNGERS AS SHOWN IN THE FIGURE*

Frequency (Gc)	Minimum Attenuation (dB)	Notes
9.26	2.0	3-dB bandwidth ≈ 13.9 Mc
11.60	58	Spurious response
12.17	56	Spurious response
13.17	57	Spurious response
13.32	58	Spurious response
13.37	58	Spurious response
14.17	63	Spurious response
14.22	63	Spurious response
15.08	61	Spurious response
15.11	54	Spurious response
15.40	67	Spurious response
16.89	64	Spurious response
17.78	49	Spurious response
18.65	45	Spurious response
21.95	44	Spurious response

* Measurements were made from 7.4 to 22 Gc, and *the spurious responses listed are those which had a minimum attenuation of roughly 60 dB or less.* The attenuation was very high between responses. The input and output guides were oriented at 45° with respect to each other.

Conclusions

The use of trapped-mode resonators in microwave band-pass filters was demonstrated to provide a means for obtaining a prescribed pass band, with very broad stop bands. It appears that by redesigning the conventional resonators at the ends of the filter so as to make their cavity proportions more dissimilar, the resonances in the conventional resonators could be further separated in frequency, and as a result the minimum stop band attenuation could be maintained at an even higher level. Although this point has not been investigated, it is probable that the power-handling ability of filters of this type will be the same as that for conventional filters using circular TE_{011}-mode resonators [13].

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Design of Band-Stop Filters in the Presence of Dissipation

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Abstract—The insertion loss vs. frequency characteristic of equal-element band-stop filters is derived for large as well as small degrees of dissipation, and for any number of resonators. These results are presented as curves for one through eight resonator filters.

The equal-element band-stop filter, for small dissipation, is shown to have the lowest pass-band loss for a specified stop-band characteristic of all possible filters that can be represented by a low-pass prototype. Design procedures and examples are explained for waveguide and TEM band-stop filters. This includes selecting the optimum number of resonators, the resonator lengths, and the coupling reactances.

Experimental results on *C*-band waveguide and UHF coaxial filters are presented; the results are in good agreement with the theory. This approach makes possible complete prediction of the filter response and results in lower pass-band loss than could be obtained with previously used approaches.

I. Introduction

Design procedures available for microwave band-stop filters [1], [2] assume lossless circuit elements or consider dissipative effects as perturbations on lossless designs. In many narrow-band applications, these assumptions are not valid; techniques that include large, as well as small, dissipative effects in the design are desirable if close control of the response is to be obtained. This paper analyzes the equal-element (periodic) band-stop filter, allowing for lossy circuit elements. The equal-element response was selected because it minimizes dissipation loss in the pass band. An analysis using a lossy low-pass prototype is described. The insertion loss has been determined as a function of v (a normalized frequency variable), with u (a normalized dissipation factor) as a parameter, for any number of elements. These results are plotted for $n=1$ through 8 (with n the number of resonant circuits in each filter). These curves form the basis of a design procedure by which the required susceptance or reactance slope parameters can be determined. At this point in the procedure, the computation of resonator lengths and coupling geometry follows directly from a procedure given by Young et al. [1]. Examples of waveguide and coaxial filters designed according to this method agree closely with the theory presented.

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